Luca Capogna

A gradient flow of diffeomorphisms for L^p dilation

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Abstract: Motivated by Ahlfors approach to study the Grotsch problem for extremal quasiconformal mappings we derive a nonlinear evolution system of PDE which is the gradient flow for the L^p norm of the outer dilation with $1 < p < \infty$. We prove short time existence, derive evolution equations for associated geometric quantities and study the asymptotic case $p \to \infty$. This is an ongoing project, joint with Andy Raich (UARK).

Giovanna Citti

???

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Abstract: ....

Katrin Fässler

Minimal distortion and modulus of curve families

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Abstract: A fundamental problem in the classical theory of quasiconformal (QC) maps in the complex plane is the identification of extremal mappings. These are homeomorphisms which minimize the L^∞-norm of the distortion within a given class of QC maps between planar domains. An analogous question can be posed for the larger class of finite distortion maps, where a mean distortion functional has to be minimized instead of the maximal distortion.

We present a method by modulus of curve families to explore such minimization problems in the complex plane and outline how this approach could be used to study extremal QC maps in the Heisenberg group.

This is joint work with Zoltán Balogh and Ioannis Platis.

Bruno Franchi

Maxwell's equations in Carnot groups

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Abstract: In this talk we present a geometric formulation of Maxwell's equations in Carnot groups (connected simply connected nilpotent Lie group with stratified Lie algebra of step κ) in the setting of the intrinsic complex of differential forms defined by M. Rumin. We show that these equations are invariant under the action of suitably defined Lorentz transformations, and we prove the equivalence of these equations with differential equations "in coordinates". Moreover, we analyze the notion of "vector potential", and we show that it satisfies a new class of 2κth order evolution differential equations.

Valentino Magnani

Sobolev surfaces and contact equations

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Abstract: In the first Heisenberg group, we discuss the nonexistence of horizontal Sobolev surfaces. Our approach is based on the nonexistence of weak solutions to the corresponding system of contact equations. This fact along with the area-type formula for surfaces shows the nonexistence of Sobolev surfaces whose intrinsic Hausdorff dimension equals two. Different notions of Sobolev surface will be discussed.

Roberto Monti

Rearrangements in metric spaces

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Abstract: We discuss some notions of rearrangement for functions and sets in the setting of a metric space with a measure. We also refer to the problem of rearrangement in the Heisenberg group.

Daniele Morbidelli

Classification of CR mappings in a class of real hypersurfaces in complex space.

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Abstract: We discuss the problem of classifying local CR mappings between strictly pseudoconvex hypersurfaces in $\mathbb{C}^{n+1}$. In particular we introduce some tools based on the analysis of classical differential invariants going back to Tanaka, Webster and Chern. Our main application concerns a class of generalized ellipsoids, where we classify all local CR mappings. (Joint with R. Monti)

Francesco Montefalcone

Local Monotonicity and Isoperimetric Inequality on Hypersurfaces in Carnot groups

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Abstract: We generalize to the setting of k-step Carnot groups an isoperimetric inequality, involving the mean curvature of the hypersurface, due to Michael and Simon, and Allard, independently. These results can be found in the recent preprint: 'Isoperimetric, Sobolev and Poincar inequalities on hypersurfaces in sub-Riemannian Carnot groups', available on Arxiv at: http://arxiv.org/pdf/0910.5656.

Alessandro Ottazzi

The Liouville theorem for Carnot groups

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Abstract: Using Tanaka prolongation theory, we show that 1-qc mappings on a domain of a Carnot group, other than $\mathbb{R}$ or $\mathbb{R}^2$, all come as restriction of the action of some finite dimensional Lie group. First we interpret the 1-qc condition at the infinitesimal level, then we show how the Tanaka construction intervenes to describe the space of 1-qc vector fields. The last step is integration to the map level. In particular, we show the calculations in the case of the Heisenberg group. This work is in collaboration with Ben Warhurst.

Pierre Pansu

Conformal Hölder exponents

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Abstract: Riemannian geometry suggests to define the conformal Hölder exponent of a map f between metric spaces X and Y as the supremum of a's in $[0,1]$ such that f is C^a with respect to some metric in the quasisymmetric gauge of Y. And to find the optimal conformal Hölder exponent of homeomorphisms between Carnot groups. Conformal dimension gives a lower bound, but unsharp.

Kirsi Peltonen

Quasiregular dynamics in the Heisenberg group

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Abstract: We report on recent developments of a higher dimensional real counterpart of the iteration theory of rational functions in the extended complex plane. A subclass of quasiregular mappings, called uniformly quasiregular mappings (UQR) have been studied first in [1] acting on the Riemann sphere and further for example in [2] acting on a compact Riemannian manifold so that all the iterates of the mapping are K-quasiregular for fixed distortion K, independently of the number of iterates.

We describe some basic constructions that can be extended further to the sub-Riemannian setting in the Heisenberg group. Moreover, we construct a non-injective uniformly quasiregular mapping g acting on the one point compactification of the Heisenberg group equipped with a sub-Riemannian metric. We further show that there exits a measurable horizontal conformal structure which is equivariant under the semigroup Γ generated by g. This is equivalent to the existence of a equivariant CR structure. This fact is interesting also from the point of view of several complex variables since it was known already for Poincaré [3] that the only semigroup of CR maps with respect to the standard CR structure must be restrictions to the sphere of a subgroup of the conformal automorphisms of the unit ball in the Euclidean space of two complex variables.

Joint work with Zoltán Balogh and Katrin Fässler (Bern University).

References

[1] T. Iwaniec and G. Martin *Quasiregular semigroups*, Ann. Acad. Sci. Fenn. **21,** No. 2 (1996), 241-254.

[2] K. Peltonen, *Examples of uniformly quasiregular mappings*, Conform. Geom. Dyn. **3,** (1999), 158-163.

[3] H. Poincaré, *Les fonctions analytiques de deux variables et la représentation conforme*, Rend. Circ. Math. Palermo **II** No. 23 (1907), 185-220.

Cornel Pintea

Size of tangencies to non-involutive distributions

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Abstract: An upper estimate is given for the Hausdorff dimension of the the tangency set of n-dimensional C^2-submanifolds of $\mathbb{R}^{n+m}$ relative to non-involutive distributions of rank n. The work is a continuation of [1], in which the author studied the case of the horizontal distribution in the Heisenberg group.

Based on a joint work with Zoltán Balogh and Heiner Rohner.

References

[1] Zoltán M. Balogh, *Size of characteristic sets and functions with prescribed gradient*, J. Reine Angew. Math., **564** (2003), 63-83.

Sverine Rigot

Monge's transport problem in non-riemannian spaces

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Abstract: The classical Monge's transport problem refers to the problem of moving one distribution of mass onto another as efficiently as possible where the efficiency is expressed in terms of the average distance transported. I will present in this talk a solution to Monge's transport problem between two compactly supported Borel probability measures in the Heisenberg group equipped with its Carnot-Caratheodory distance assuming that the initial measure is absolutely continuous with respect to the Haar measure of the group (joint work with L. De Pascale). The Heisenberg group is taken here as an illustrative explicit example of non-riemannian space and I shall also discuss extension of the techniques used in this work to other metric spaces.

Jeehyeon Seo

Bi-Lipschitz embeddability of the Grushin plane into Euclidean space

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Abstract: Many sub-Riemmanian manifolds like the Heisenberg group do not admit bi-Lipschitz embedding into any Euclidean space. In contrast, the Grushin plane admits a bi-Lipschitz embedding into some Euclidean space. This is done by extending a bi-Lipschitz embedding of the singular line, using a Whitney decomposition of its complement.

Raul Serapioni

Characterizations of rectifiable sets in $\mathbb{H}^n$

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Abstract: According to Federer's definition, k-dimensional rectifiable sets in $\mathbb{R}^n$ are contained, up to a negligible set, in the countable union of Lipschitz images of subsets of $\mathbb{R}^k$. In Euclidean spaces it is equivalent to use coverings with countable unions of k-dimensional C^1 submanifolds or to require the almost everywhere existence of approximate tangent spaces or of tangent measures. Also in groups it is possible to follow the pattern of the three definitions, provided we have good intrinsic notions of Lipschitz maps or of C^1 submanifolds or of approximate tangent subgroups and tangent measures. But the complete equivalence of the three definitions is still an open question. We discuss some partial equivalence of the three definitions inside Heisenberg groups.

Francesco Serra Cassano

Non-parametric minimal surfaces in Heisenberg groups: existence and local boundedness.

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Abstract: In the setting of Heisenberg groups, endowed with their sub-Riemannian structure, two notions for the graph of a function have been introduced. Putting particular emphasis on the so called intrinsic graphs, we define the measure of a graph and prove the existence of minimal graphs. As a first step towards the regularity of minimal surfaces in Heisenberg groups, we show that minimal graphs must be locally bounded. This is joint work with D. Vittone .

Marc Troyanov

Conformal mappings in Finsler geometry

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Abstract: ...

Davide Vittone

Isodiametric sets in the Heisenberg group

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Abstract: In this talk we focus on isodiametric sets in the Heisenberg group, i.e. sets maximizing the volume measure among those with fixed diameter. We first show a Lipschitz regularity result for the boundary of such sets. We are able to solve the isodiametric problem in the restricted class of rotationally invariant sets, where the solution is given by the (Euclidean) convexification of CC-balls. A nonuniqueness result is also shown. This is joint with G. P. Leonardi and S. Rigot.

Ben Warhurst

Rigidity classification of Carnot groups in the class of C^2 maps via Tanaka theory

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Abstract: This aim of this talk is to show how the prolongation theory of Tanaka combines with the prolongation theory of Singer and Sternberg to give a precise rigidity classification of Carnot groups at the level of C^∞ contact vector fields. We then show that this classification extends to the class of C^2 contact maps. This is joint work with Alessandro Ottazzi.

Stefan Wenger

Lipschitz extensions to jet space Carnot groups

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Abstract: Lipschitz extension problems have received considerable attention for a long time. Not much is known, however, for Carnot-Caratheodory space targets. In this talk, I will present new Lipschitz extension and non-extension results in the case that the target space is a jet space. Recall that jet spaces give a model for a certain class of Carnot groups, including the (higher) Heisenberg groups, the Engel group, and the filiform groups. Based on a collaboration with Severine Rigot and a collaboration with Robert Young.

Roger Züst,

Integration of Hölder forms and currents in snowflake spaces?

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Abstract: For an oriented n-dimensional Lipschitz manifold M we give meaning to the integral $\int_M f\, dg_1 \wedge \cdots \wedge dg_n$ in case the functions $f, g_1, \ldots, g_n$ are merely Hölder continuous of a certain order by extending the construction of the Riemann-Stieltjes integral to higher dimensions. More generally, we show that for $\alpha \in (\frac{n}{n+1}, 1]$ the n-dimensional locally normal currents in a locally compact metric space (X, d) represent a subspace of the n-dimensional currents in (X, d^α). On the other hand, for $n \geq 1$ and $\alpha \leq \frac{n}{n+1}$ the vector space of n-dimensional currents in (X, d^α) is zero.